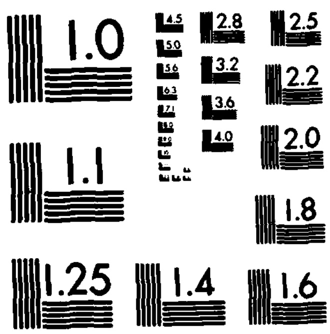
AD-A122 913 NUMERICAL ANALYSIS AND NONLINEAR MECHANICS(U) 1/1
MASSACHUSETTS INST OF TECH CAMBRIDGE DEPT OF
MATHEMATICS W G STRANG DEC 82 MIT-89611
UNCLASSIFIED ARO-17256.11-MA DAAG29-80-K-0033 F/G 12/1 NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ARO 17256.11-MA

(12)

AD A122913

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Report 1	2. GOVT ACCESSION NO. AD-A122913	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) NUMERICAL ANALYSIS AND NONLINEAR MECHANICS		5. TYPE OF REPORT & PERIOD COVERED Final Report 07·01·80--09·30·82
		6. PERFORMING ORG. REPORT NUMBER M.I.T.-89611
7. AUTHOR(s) Professor W. Gilbert Strang		8. CONTRACT OR GRANT NUMBER(s) DAAG29 80 K 0033
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mathematics Massachusetts Institute of Technology Cambridge, MA 02139		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE December 1982
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

JAN 0 3 1983

E

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

optimal design, plasticity, mechanics, numerical analysis, optimization.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We have begun an analysis of the optimal design of structure in mechanics and biomechanics, with emphasis on materials which do not satisfy Hooke's law. Our research has been partly numerical and partly analytical; the optimal design problem is highly nonlinear with respect to variations in the geometry, and this has meant that the construction of efficient algorithms is unusually difficult. They depend on the formulation of the optimization problem, and we

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DTIC FILE COPY

83 01 03 027

20. continued

have made progress in this area--to replace a nonconvex cost function by a convex cost. In the case of multiple loadings our new formulation is polyconvex, and may be the first case of its type to yield explicit minimization of functionals of a Jacobian matrix.

REPORT 1: FINAL REPORT

NUMERICAL ANALYSIS AND NONLINEAR MECHANICS

CONTRACT DAAG29 80 K 0033

U. S. ARMY RESEARCH OFFICE

Professor W. Gilbert Strang
Department of Mathematics
Massachusetts Institute of Technology
Cambridge, MA 02139

December 1982

Summary of Results

This research has yielded progress in the area of optimal design, which involves new mathematical questions in the formulation of a correctly posed optimization problem. The initial statement of the problem is difficult both analytically and numerically; in many cases the cost function is nonconvex. Working jointly with Robert Kohn we have found an equivalent convex problem for the case of a single loading, and our current research has led to the calculation of an equivalent polyconvex problem for the case of multiple loadings.

Our results in these different areas will be summarized separately:

1) For cylinders in torsion or in shear, we have determined the minimal structure within a given region that can achieve a specified torsional rigidity or support a prescribed load. By constructing explicit solutions we established a class of problems for numerical analysis, and we developed analytical methods which extend to other geometrics and other materials.

2) We prepared and tested a code to link the quasi-Newton methods in nonlinear optimization to the existing programs for finite element analysis. Our package includes two update techniques, the BFGS rank two update for positive definite symmetric systems (convex optimization) and the Broyden rank one update for unsymmetric systems with convective terms. Both have been requested by engineers. Dr. Engelman continues to work on the development of

a general fluid dynamics--finite element code, which is an essential tool in scientific computing.

3) The max flow-min cut theorem of operations research was extended to the continuous case, with flows through a domain and local capacity constraints on the velocity field. A parallel development by Iri has led to applications in the analysis of traffic flow.

4) In joint work with Iserles we completed a study begun much earlier, by establishing the upper limits on the accuracy of a stable method for hyperbolic equations. The accuracy is restricted by the choice of meshpoints, and high accuracy cannot be achieved in the full region permitted by the Courant condition on flow speeds. We studied both explicit methods and a new class of implicit Pade schemes.

5) Ballistic penetration problems lead to regions of extremely rapid straining and to loss of monotonicty in the stress-strain curve. In a report at Aberdeen Proving Grounds we discussed progress on the understanding of shear bonds. We reported also on numerical experiments with the equation $u_t = (\sigma(u_x))_x$, giving a bounded solution with oscillatory derivatives in the region where $\sigma' < 0$.

We separate for special mention two continuing research projects which have come from the original results on optimal design:

6) For two or more loads, the mathematics of relaxation (or homogenization) is much more difficult: the relaxed form is no longer convex in the stresses σ, τ. Nevertheless it is lower semicontinuous and a minimum is achieved.

This problem was studied abstractly by Morrey, who found a property that replaces convexity in the relaxed integrand.

We have computed the first example that fully illustrates this theory. It arises in elastic design for given compliance:

$$\inf \int (1_{\{|\sigma|+|\tau|\neq 0\}} + |\sigma|^2 + |\tau|^2)\, dx = \min \int G(\sigma,\tau)\, dx.$$

It is the integrand G which we found, and it is polyconvex--a convex function of σ, τ, and $D = \sigma^{\perp}\cdot\tau$. We now have two ways to compute G. One uses the bounds found by Tartar on the moduli of a composite elastic material, given the fraction ρ of the original material used to construct the composite. Our method is to consider all possible composites near each point, so that

$$1 + |\sigma|^2 + |\tau|^2 \to \rho + (A\sigma,\sigma) + (A\tau,\tau)$$

where A gives the compliance of the composite. Then we minimize over all $A(\rho)$ allowed by Tartar, and over all ρ.

Our second method is more general, but the computations are again not easy. It finds the "polyconvex hull" of the graph of the original integrand.

We have just begun on these ideas. They are certain to lead further--to a duality theory, if possible, and to a deeper understanding of nonconvex variational problems in R^n.

7) Jointly with the orthopaedic research group at Beth Israel hospital, we are studying Wolff's law for the structure of bone fibers: their density and direction are determined by the imposed loads. The first numerical experiments show agreement between the features of a theoretically optimal design and the actual distribution within bones. We plan to compare their stereological description and the output from finite element analyses with the optimal designs computed from further numerical optimizations. The code approximates the Michell theory of optimal trusses, and leads in two dimensions to orthogonal nets of Hencky-Prandtl type. The orthogonality of bone fibers can be studied experimentally; if confirmed, the three-dimensional analysis will lead to new questions in optimal design. We believe there are applications of both 6) and 7) to the construction of composite materials.

Participating Scientific Personnel

1. Professor W. Gilbert Strang
   Department of Mathematics
   Massachusetts Institute of Technology

2. Dr. Michael Engelman
   Research Associate, M.I.T.
   Appointed to Mathematics Department, I.I.T.

Publications

1. The application of quasi-Newton methods in fluid mechanics, with M. Engelman and K. J. Bathe, Int. J. Num. Meth. Eng. 17 (1981) 707-718.

2. A problem in capillarity and plasticity, with R. Temam, Nondifferential and Variational Techniques in Optimization, D. C. Sorenson, R.J.B. Wets, eds., Mathematical Programming Study 17 (1982) 91-102.

3. Optimal design for torsional rigidity, with R. Kohn, Proc. Int. Symp. on Mixed and Hybrid Finite Element Methods, Atlanta (1981).

4. Optimal design of cylinders in shear, with R. Kohn, The Mathematics of Finite Elements and Applications IV, J. R. Whiteman, ed., Academic Press (1982) London.

5. The width of a chair, American Math. Monthly 89 (1982) 529-534.

6. Structural design optimization, homogenization, and relaxation of variational problems, with R. Kohn, Proc. Conf. on Disordered Media, Lecture Notes in Physics 154, Springer-Verlag (1982) New York.

7. Hencky-Prandtl nets and constrained Michell trusses, with R. Kohn, Conf. on Optimum Structural Design, Tucson (1981), to appear in Computer Methods in Applied Mechanics and Engineering.

8. The optimal accuracy of difference schemes, with A. Iserles, Transactions Amer. Math. Society, to appear.

9. Duality in the classroom, submitted to American Mathematical Monthly.

10. Maximal flow through a domain, Mathematical Programming (1982).

11. Optimal design and convex analysis, with R. Kohn, in preparation.

12. L^1 and L^∞ approximation of vector fields in the plane, Proc. U.S.-Japan Seminar on Nonlinear Partial Differential Equations, Tokyo (1982), to appear.

13. Notes on softening and local instability, with M. Abdel-Naby, ARO Workshop on Mechanics, Springer Lecture Notes in Engineering, to appear.

END

FILMED

2-83

DTIC